

# Modeling Wind Wave Evolution from Deep to Shallow Water

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## **LONG-TERM GOALS**

Ocean waves are an important aspect of upper ocean dynamics, in particular on the shallow continental shelves and in coastal areas. The long-term objective of this work is to advance wave modeling capability in such coastal areas by improving model representations of effects associated with nonlinearity, inhomogeneity, and dissipation.

## **OBJECTIVES**

The specific objectives of the present work are 1) to develop and implement an efficient and scalable approximation for the nonlinear quadruplet source term, 2) to develop and implement a generalized nonlinear source term that is accurate in water of arbitrary depth, 3) to develop an improved nonlinear closure for triad nonlinear interactions in shallow water, and 4) improve representations of dissipation by wave breaking and wave-bottom interactions in shoaling waves.

## **APPROACH**

Modern, third-generation (3G) wave models are based on an action balance (or radiative transfer) equation, which describes the transport of wave energy (or action) through a slowly varying medium and time. In Lagrangian form (for convenience) this balance equation can be written as

$$\frac{dN(\mathbf{k})}{dt} = S_{in}(\mathbf{k}) + S_{ds}(\mathbf{k}) + S_{sc}(\mathbf{k}) + S_{nl}(\mathbf{k}) \quad (1)$$

where  $N(\mathbf{k})$  is the wave action at wavenumber vector  $\mathbf{k}$  and  $t$  is time. The forcing terms on the right-hand side are referred to as source terms and account for the input of energy by the wind ( $S_{in}$ ), spectral redistribution of energy through scattering by seafloor topography ( $S_{sc}$ ) or through nonlinear wave-wave interactions ( $S_{nl}$ ), and dissipation of wave energy ( $S_{ds}$ ) through e.g. breaking or bottom friction.

In this study we will develop and improve the source terms for nonlinear interactions  $S_{nl}$  and energy dissipation  $S_{ds}$ , to account for effects of finite depth and shallow water, and to ensure a consistent and smooth model representation of wave evolution from deep to shallow water.

### **Nonlinearity**

We will develop an efficient method for the evaluation of the nonlinear source term, allowing for greater efficiency and accuracy in operational use. To allow modeling of wave propagation from deep to shallow water, we will modify the nonlinear source term to account for changes in relative water depth [Janssen *et al.* 2006], and develop an improved closure approximation for nearshore wave propagation [Janssen, 2006].

### **Dissipation**

We will develop and test improvements to wave dissipation parameterizations through detailed comparisons of model results to laboratory and field observations in a wide range of conditions.

### **Field data**

We will analyze, prepare, and disseminate selected field experimental data sets, collected by the PI's, to the project teams for the purpose of validation and calibration of new model developments.

## WORK COMPLETED

### Development of a Lumped Quadruplet Approximation (LQA)

A scalable parameterization of non-linear four-wave interactions is being developed to bridge the gap between time consuming exact methods and the fast but inaccurate Discrete Interaction Approximation (DIA). The focus in this work is on developing a consistent approximation of the complete interaction manifold based on the WRT method. We follow two main lines of development for achieving this.

The first main development is to improve numerical efficiency and to limit the integration space by applying higher-order integration methods, filtering methods, simplified interpolation procedures, and re-sampling of points on the locus. This will result in a more practical, albeit still computationally intensive, version of the WRT method for application in operational wave models.

The second main development is what we refer to as the Lumped Quadruplet Approximation (LQA), in which discrete contributions on the locus are treated as individual wave number configurations, which can be handled by the Generalized Multiple DIA Approaches (see e.g. Van Vledder, 2001; Tolman, 2012). Using this technique, various optimized sets of wave number configurations can be derived (see figure 1).

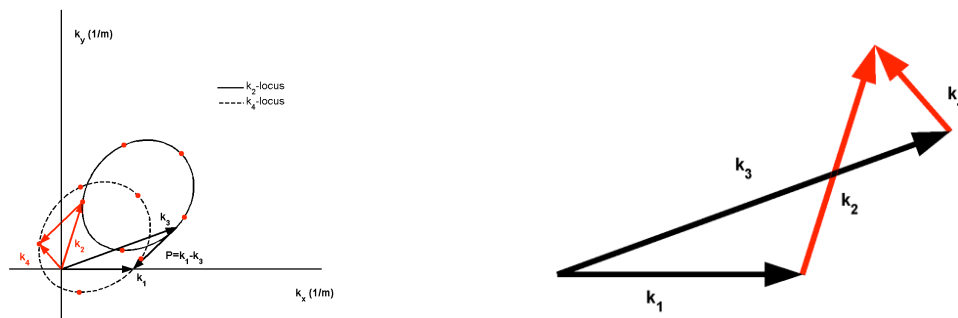


Figure 1 Wave number configuration based on discretization of integration along a locus in wave number space.

We are working on developing a comparison between the three main approaches to evaluate the exact four-wave interactions: 1) the WRT method as implemented by Resio and Perrie, and Van Vledder, 2) Masuda's method implemented in the RIAM method by Komatsu, and 3) the method of Lavrenov implemented as the GQM by Benoit and Gagnaire-Renou (see e.g. Van Vledder, 2012). Although each of these methods aims to solve the same integral, it is not clear if they actually produce the same answers. This study is done in collaboration with Drs Hashimoto (RIAM) and Benoit (GQM).

## Analysis and dissemination field observations.

We have disseminated the ONR NCEX (Nearshore Canyon Experiment) field observations to the NOPP teams. These observations are particularly well suited to validate model representation of refraction and wave focusing over extreme topography.

We have also completed a comprehensive analysis and quality control of a data set of wave propagation across the Louisiana mud shelf. This dataset will be shared with the NOPP teams and will provide a unique set of observations of wave propagation over mud and the effects of larger-scale circulation and sediment dynamics on coastal wave evolution. The analysis findings will contribute to understanding of dissipation over mud in operational wave models.

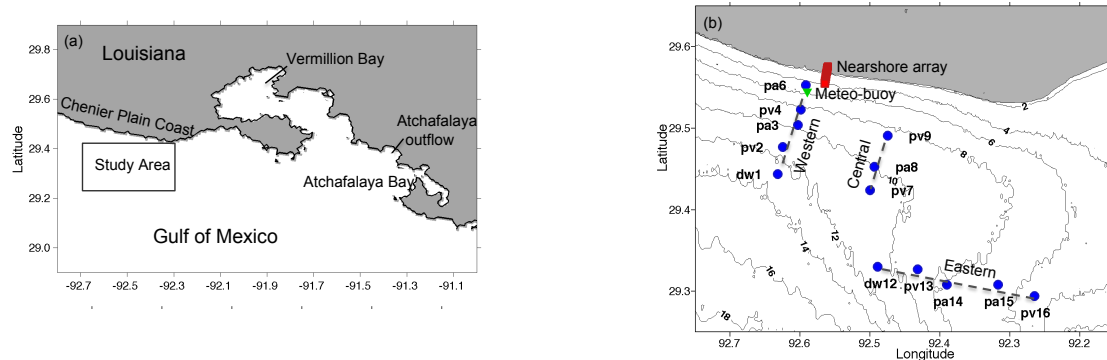


Figure 2 Overview of experimental area (left panel) and sensor locations (right panel, in blue) for the 2008 Louisiana Waves-over-Mud MURI experiment.

## Testing of a one-point closure shallow-water model

We have tested a one-point closure approximation in a shallow-water wave model against laboratory observations and Monte-Carlo simulations. The purpose of these tests is to explore the validity of this efficient alternative for shallow-water wave modeling.

## Transport equations for wave correlators

We have further developed a general theory for the transport of cross-correlations in random wave fields. These correlations are essential to model inhomogeneous and non-Gaussian effects in natural wave fields. We have studied the effects of two-wave correlators in focusing wave fields and in regions of diffraction. This work forms the basis for a new, isotropic description of linear and nonlinear wave dynamics that includes inhomogeneous wave fields, but is compatible with the action balance generally used in operational wave models.

## RESULTS

### Development LQA method

From a two-week hindcast study of wave conditions in Lake Michigan (completed in collaboration with Dr Erick Rogers), it is found that more accurate approximations of four-wave interactions in operational wave models affect the spectral narrowness (in frequency space). There is considerable variation in frequency narrowness between cases where four-wave interactions are computed with the WRT or the DIA (see figure 3). This is important to understand and has implications for the use of more exact methods (or better approximations thereof) in operational wavemodels, and the computation of bulk parameters from model results to characterize the spectral shape.

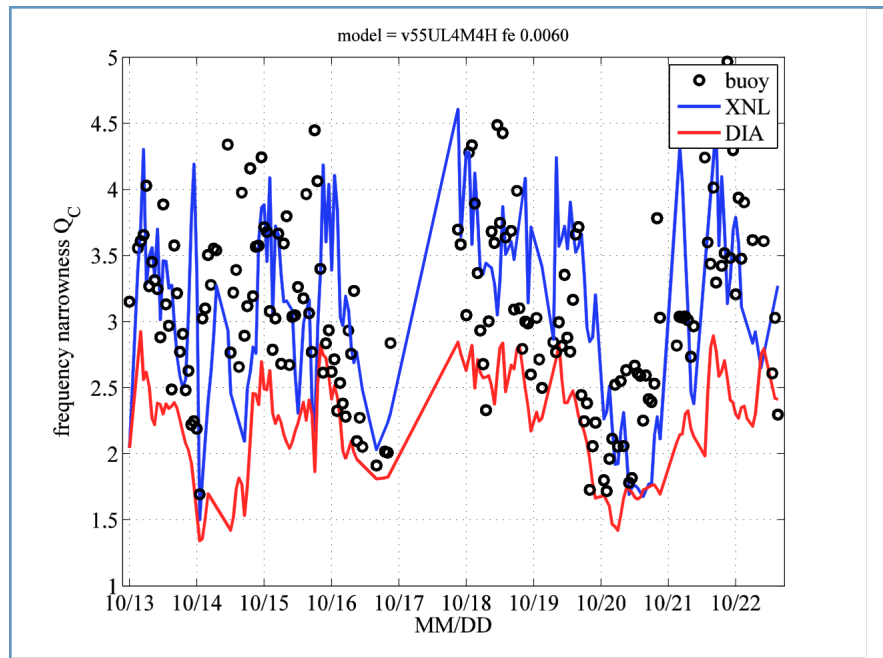


Figure 3 Comparison of measured and computed spectral narrowness using SWAN and two methods for computing the non-linear interactions. Period 13-22 October 2002, Lake Michigan (from Rogers and Van Vledder, 2012).

### Analysis and dissemination of field observations

Our analysis of a new dataset collected during the ONR Waves over Mud MURI, shows that the effects of mud on wave evolution can be time varying and episodic. During onshore swell conditions, the observations show, as expected, enhanced dissipation. However, an even more pronounced effect of the muddy bottom on wave evolution was seen during fetch-limited, wind-forced conditions. The presence of mud was seen to suppress wave growth during such conditions. From satellite observations (see figure 4), the observed episodes of strong mud-induced effects on wave development were related to larger-scale plume dynamics originating in the Atchafalaya Bay system and extending westward into the research area during easterly wind conditions.

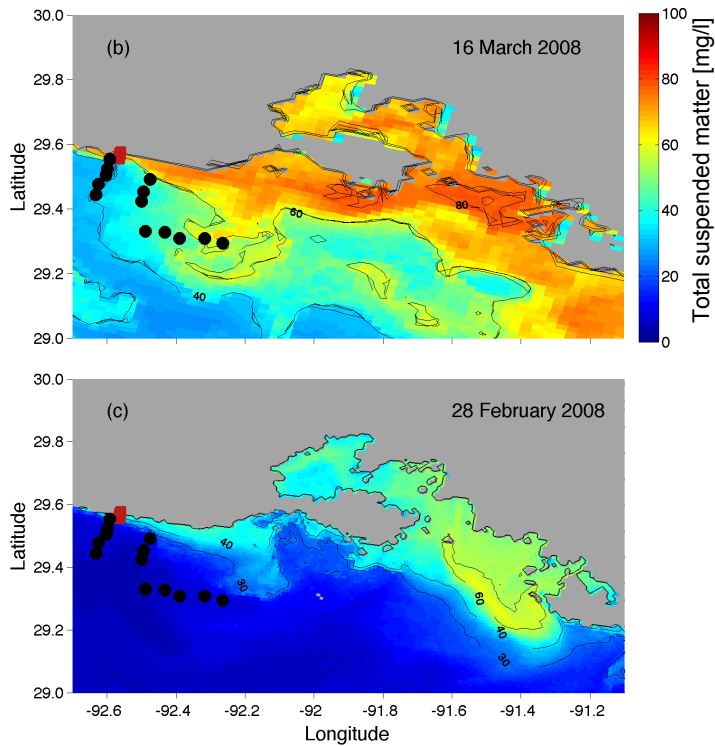


Figure 4 Estimate of total suspended-matter concentrations (from Modis 250 imagery) on (b) March 16, and (c) February 28. High concentrations are red, low concentrations are blue (color scale on right). Grey shading is land. Instrument locations are shown with black circles (shelf stations) and red squares (shoreward stations). The strong variability in sediment concentrations due to changes in wind are shown to strongly affect dissipation and wave growth characteristics in the experimental area.

### Analysis and modeling of nonlinearity and dissipation in shoaling waves

Validation of a new shallow-water model using a quadratically weighted dissipation function (e.g. Kaihatu et al. 2007) and a quasi-empirical relaxation model shows excellent agreement with observations (see figure 5). This shows that a relatively simple and quasi-empirical parameterization of the nonlinear relaxation length scales can indeed capture the principal nonlinear dynamics in shoaling gravity waves at a fraction of the computational cost of a more complete two-point closure method.

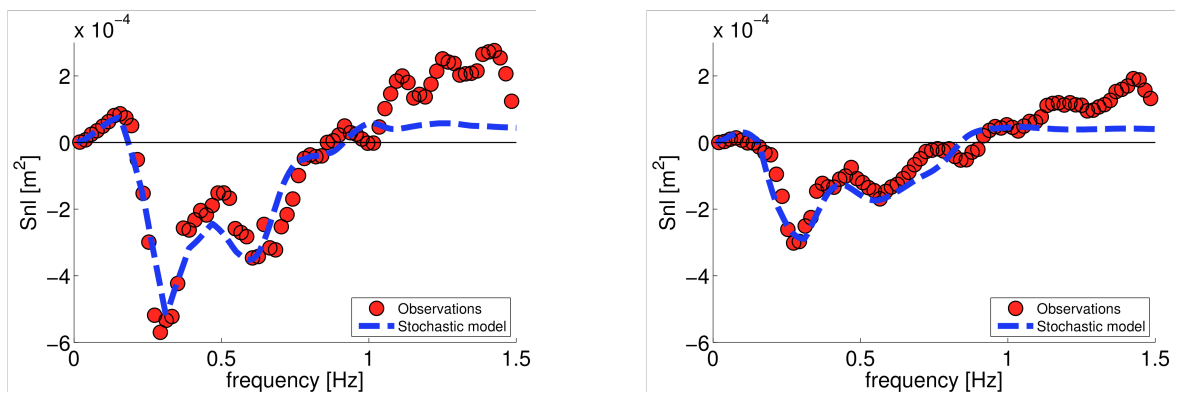


Figure 5. Comparison of observations (red circles) by Boers (1996) and model results with stochastic shallow-water model (blue dashed line) of evolution of nonlinear transfer at  $x=20\text{m}$  (left panel), just over the outer bar, and  $x=25\text{m}$  (right panel), well inside the surf zone. In the laboratory experiments by Boers (1996), the beach mimics a bar-trough profile with the still-water shoreline at about 30m from the wavemaker.

## Transport equations for inhomogeneous wave fields

Evaluation of the new transport equation for inhomogeneous (but Gaussian) wave fields, which forms a natural extension of the radiative transfer equation (see Smit & Janssen 2012), shows very promising results. With the new model it is possible to accurately resolve coherent interferences in random waves (see figure 6). This new approach for stochastic modeling forms the basis for the derivation and implementation of an isotropic model for the transport of three-wave correlations (bispectrum) through variable media, which is critical to extend stochastic models to shallow water.

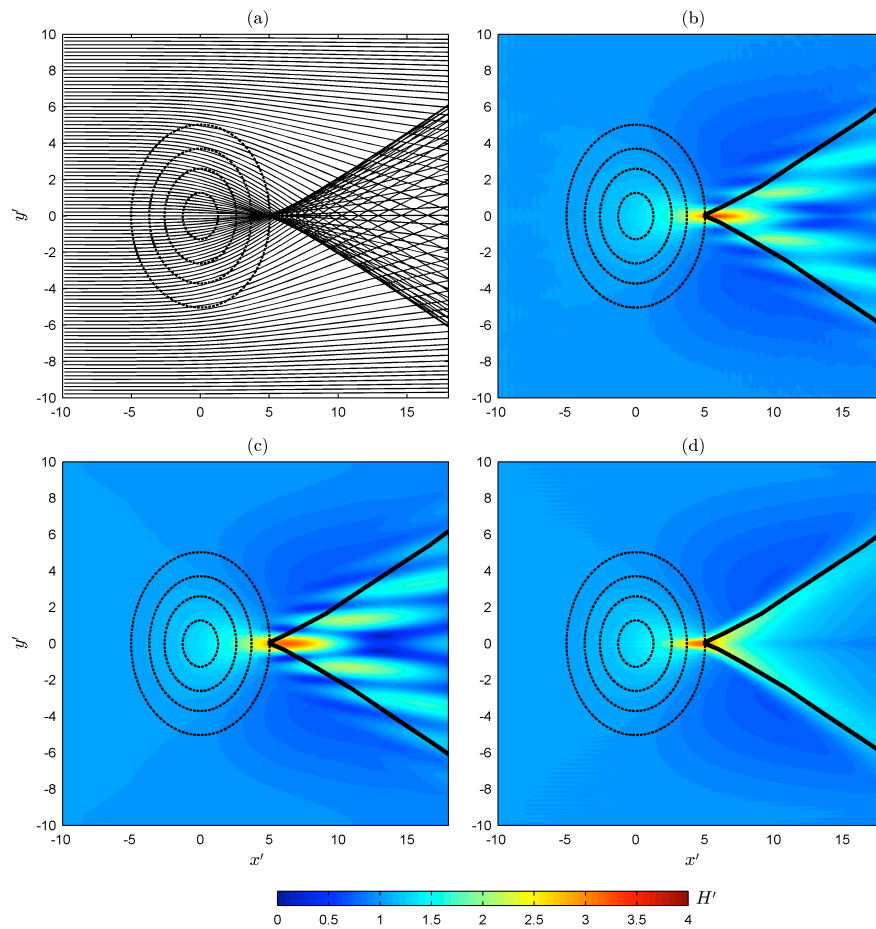


Figure 6 Shown are different approximations to wave focusing over a shoal. Narrow-band waves are propagating from left to right; depth contours are shown as thin black lines. Panel (a) shows ray pattern of geometrical optics, with well-defined focal point and caustic region behind shoal. Panel (b) shows Monte Carlo simulations with deterministic shallow-water model, which shows coherent (refraction-diffraction) energy pattern in caustic area (caustic lines are approximately indicated by thick solid lines in panels b, c, and d). Panel (c) shows the result of the new, coherent stochastic model and panel (d) shows the predicted energy distribution by the radiative transfer equation. It is clear from the comparison that the new stochastic model accurately includes coherent patterns in the spatial distribution of wave energy, whereas the radiative transfer equation becomes inaccurate in the vicinity of the caustics.

## **IMPACT/APPLICATIONS**

### **Economic Development**

The improvements to coastal wave prediction models as developed in this project, will contribute to various economic activities on the continental shelf and the coastal zone, such as fisheries, and shipping and offshore industry. Further, the availability of better wave prediction models will benefit coastal and ocean engineering companies e.g. in the design and operation of offshore and coastal structures, and the development of coastal management strategies.

### **Quality of life**

The improvements to coastal wave prediction models as developed in this project, will improve modeling capability of coastal circulation and transport processes, which will benefit coastal recreation (more reliable knowledge of wave heights, rip currents etc), coastal management, and help mitigate pollution hazards for humans (recreation) and coastal ecosystems.

## **TRANSITIONS**

### **Economic Development**

The developments in this project will be made available as open source software and as modules to widely used operational wave models. These models are used by NOAA and other agencies involved in coastal development and management, and by many coastal and ocean engineering companies.

### **Quality of life**

The software developed within this project will be disseminated in open source models used by local and federal agencies and companies involved in coastal recreation (surf prediction, rip currents, pollution), coastal management, and mitigation of coastal hazards.



## RELATED PROJECTS

The development of transport equations for cross-correlations in random waves also contributes to the study of coastal wave-current interaction as part of the ONR Inlets and River Mouths DRI.

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